R & D NOTES

The Effect of Tube Walls on Drag Coefficients of Coaxially Placed Objects

ALEKSANDAR P. DUDUKOVIĆ

Institute for Petrochemistry, Natural Gas, Oil and Chemical Engineering
Department of Chemical Engineering
Faculty of Technology
Novi Sad, Yugoslavia

and

SLOBODAN K. KONČAR-DJURDJEVIĆ

Department of Chemical and Metallurgical Engineering
Faculty of Technology and Metallurgy
Beograd, Yugoslavia

The interest in the behavior of sphere or disk moving through a fluid goes back many years. The first recorded measurements related to sphere drag were published by Sir Isaac Newton (1729); a multitude of measurements of sphere and disk drag coefficients followed. However, as far as we know, there are hardly any quantitative data in the literature on the effect of tube walls on drag coefficients, except the results obtained by Achenbach (1974) for spheres. The aim of this investigation is to examine the effect of the ratio of object and coaxial tube diameters on drag coefficients. Experiments are performed for disks and spheres as two extremes in sphericity in order to enable the assessment of drag coefficients for objects of different sphericity.

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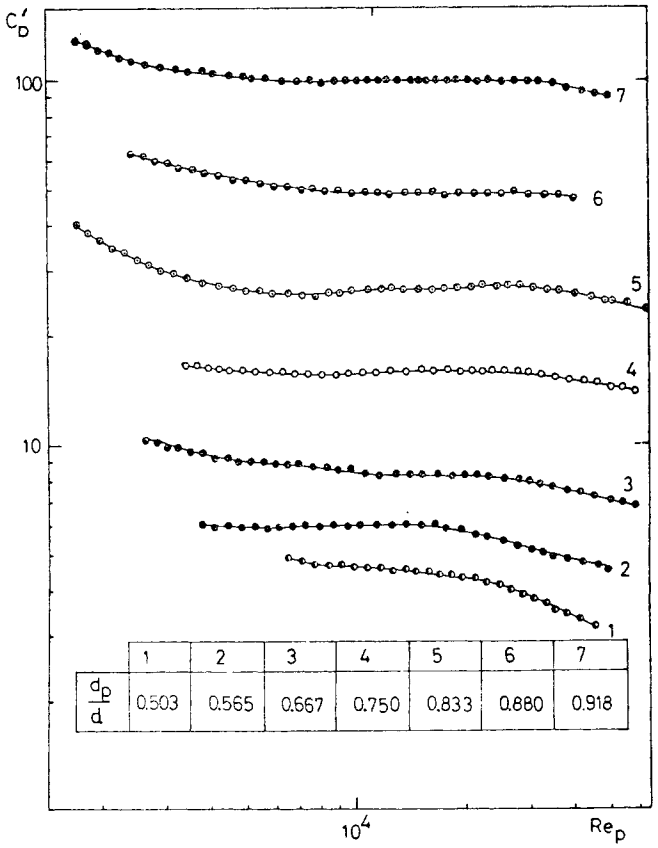
APPARATUS

The apparatus consisted of two reservoirs placed on two levels with an overflow on the upper one and a centrifugal pump enabling constant flow during the course of every experiment. Objects are placed in a vertical tube made of plexiglass of 60 mm in diameter. The object is always placed 35 tube diameters from the entrance, so that the flow in the tube was fully developed prior to reaching the obstruction. The spheres were



	1	2	3	4	5	6	7
$\frac{d_p}{d}$	0.503	0.565	0.667	0.750	0.833	0.880	0.918

Figure 1. Drag coefficient as a function of Reynolds number for a disk in a tube.

fixed by a steel shaft, 1.8 mm thick, mounted perpendicularly to the flow direction. The disk has an additional supporting rod 1.8 mm thick at the angle of 90 degrees to the shaft and also perpendicular to the flow direction. Disks diameters were as follows: 30.2; 33.9; 40.0; 45.0; 50.0; 52.8; and 55.1 mm, while those of spheres were as follows: 30.0; 34.5; 40.0; 47.6 and 51.0 mm.

The pressure drop was measured at the ends of the operating tube by "U" manometers filled with mercury, chloroform or methylbenzoate.

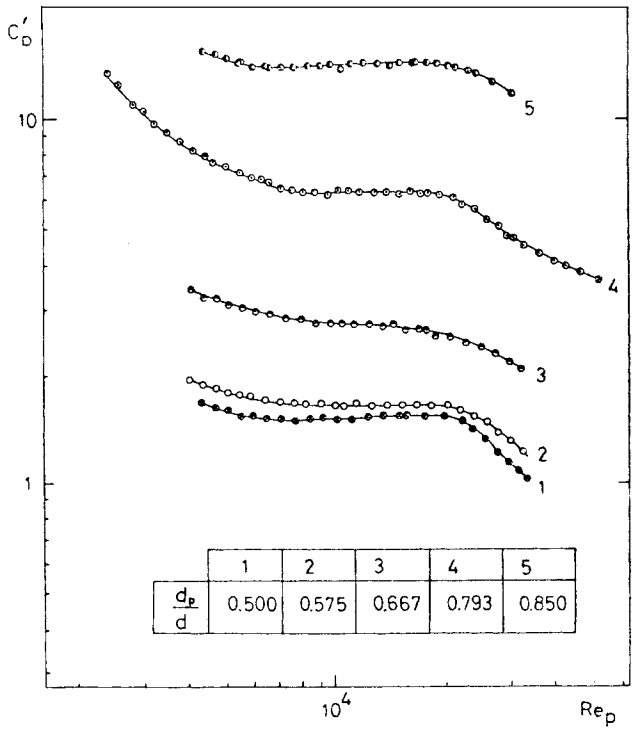
RESULTS

Drag coefficient is usually determined directly by measuring the drag force on the object. However, we proposed a formula for the drag coefficient C_D' which allows its evaluation from the pressure drop in the fluid from which the pressure drop caused by the resistance of supporting axles as well as by the resistance of the tube itself is substracted (Dudukovič, 1978):

$$C_D' = \frac{(\Delta p/\gamma)(d/d_p)^2}{w^2/2g} \tag{1}$$

The term $(d/d_p)^2$ in equation (1) takes into account that the pressure loss per unit cross-section of the object is distributed over the whole cross-section of the tube. Therefore, it is expected that the drag coefficient defined in the above manner corresponds closely to the standard one C_D, calculated directly from drag force. This seems to be confirmed by experimental results.

From the measured overall pressure drop in the tube we substracted the pressure drop caused by the resistance of supporting rods and the one which would have existed in an empty tube under same conditions of flow. When equation (1) is used to account for the pressure drop due to the supporting rods the



	1	2	3	4	5
$\frac{d_p}{d}$	0.500	0.575	0.667	0.793	0.850

Figure 2. Drag coefficient as a function of Reynolds number for a sphere in a tube.

following formula for evaluation of the pressure drop to be used for determination of C_D' of the object is obtained.

$$(\Delta p/\gamma) = (\Delta p/\gamma)_{exp} - \lambda(L/d)(w^2/2g) - (C_D)_{cyl}(w^2/2g)\frac{n\, d_a(d - d_p)/2}{(d^2 - d_p^2)\pi/4} \quad (2)$$

where $(\Delta p/\gamma)_{exp}$ is the measured pressure drop. The second term on the right-hand side of equation (2) represents the pressure drop of an empty tube under the same flow conditions, while the third term is the pressure drop caused by the supporting rods. $(C_D)_{cyl}$ is the drag coefficient for an infinite cylinder perpendicular to flow at Reynolds numbers calculated for the velocity in the annular space and rod diameter obtained from the results of Wieselsberger (1922); n is the number of supporting rods ($n = 2$ for sphere and $n = 3$ for disk). It is important to note that the third term on the right hand side of equation (2) never exceeded 5% of the final calculated value on the left hand side in the range of variables covered in this study.

The obtained results are shown in Fig. 1 for a disk and in Fig. 2 for a sphere.

DISCUSSION

Figures 1 and 2 which show the dependence of the drag coefficients C_D' on the Reynolds number ($Re_p = w\, d_p\rho/\mu$) for various ratios of object and tube diameters have some common characteristics. The drag coefficient is nearly constant and independent on Reynolds number. This is quite understandable since drag coefficients for disks and for spheres in an infinite fluid proved to be also nearly constant in this range of Reynolds number (10^3 to 10^5). However, for Reynolds numbers greater than $Re_p \approx 2{\cdot}10^4$ the drag coefficient for a sphere starts to decrease. For a single sphere however, the appearance of the critical region in the C_D curve is not expected at such small Reynolds numbers. It is possible that this drop of the drag coefficient is caused by an early transition of boundary layer on the sphere to a turbulent one due to the presence of the supporting rods. Similar phenomena of an early decrease of the drag coefficient have been observed by Bacon and Reid (1924) and were caused by an early onset of turbulence as shown later by Bailey (1974).

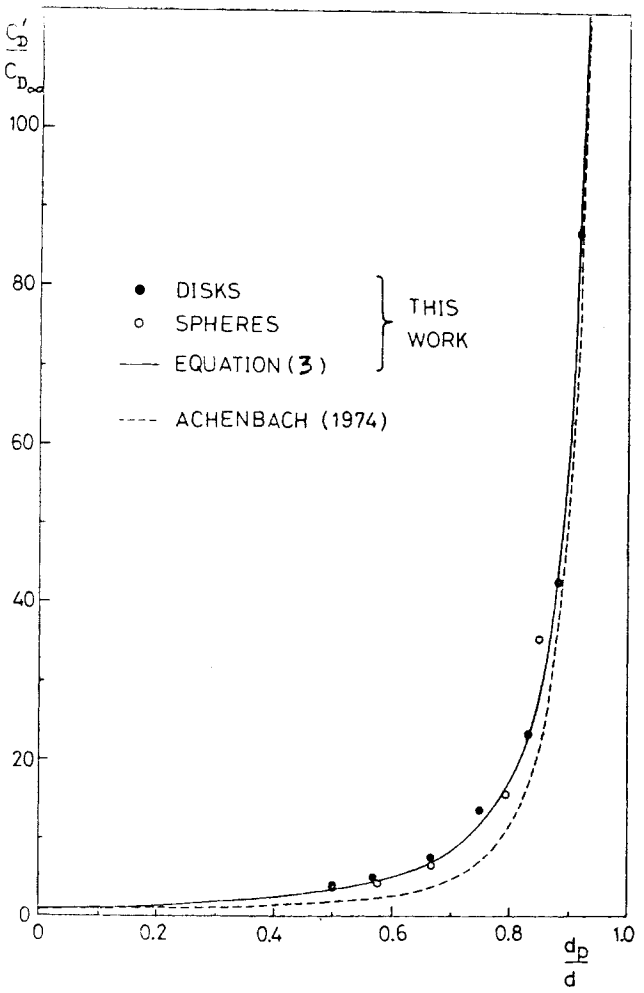


Figure 3. Correlation for a drag coefficient for disks and spheres in a tube.

In the range of the Reynolds number where the drag coefficients are approximately constant, one may find the dependence of drag coefficients on the ratio of object and tube diameters. The dependence of the ratio of drag coefficient C_D' and drag coefficient in infinite fluid $C_{D\infty}$ on the ratio of object and tube diameter d_p/d for disk and sphere at $Re_p = 2{\cdot}10^4$ is shown in Figure 3. A linear regression method has shown that the same equation correlates the results for disk and sphere. The obtained equation is as follows:

$$\frac{C_D'}{C_{D\infty}} = \frac{1}{(1 - d_p/d)^{1.78}}\ ;\ (2{\cdot}10^3 < Re_p < 2{\cdot}10^4) \quad (3)$$

whereas the correlation coefficient was $r = 0.995$. In the above formula the values for $C_{D\infty}$ are taken from the results of Bailey (1974) for the sphere and Roos and Willmarth (1971) and Wieselsberger (1922) for the disk. The fact that the same equation correlates the results for disk and sphere in the above region of Reynolds numbers leads us to believe that the same correlation holds for all other objects due to the fact that the sphere and the disk represent extremes in sphericity. It should be pointed out that equation (3) was obtained for the data outside the region in which supporting rods potentially have caused an anomalous behavior for the C_D' for a sphere and thus should be accepted as a reliable and useful tool in the region of Reynolds numbers indicated.

Our results are compared with the results of Achenbach (1974) in Figure 3. Experiments of Achenbach were performed at higher Reynolds numbers ($Re \sim 10^5$). He used a different definition for the drag coefficient and Reynolds number. When the values obtained from his correlation equation are converted into the common drag coefficient the obtained curve has the shape shown in Fig. 3.

The curves representing our correlation and the correlation by Achenbach (Fig. 3) show good agreement in general shape and trend. Both indicate relatively small increase of the drag coefficient up to $d_p/d \sim 0.5$, followed by an abrupt increase. However, Achenbach's results are markedly smaller, e.g. for $d_p/d = 0.5$ the deviation from our curve is approximately 50%, but with the increase of d_p/d these differences decrease. According to our opinion, these differences are the result of different experimental conditions which caused differences in velocity profiles. Achenbach carried his experiments by placing the spheres just at the entrance of a windtunnel, which means that the velocity profile was not fully developed. In our experiments, however, the velocity profile is completely formed and the greatest energy losses due to the presence of the object are experienced by the fluid elements with the higher kinetic energy. Therefore, the drag coefficient should be higher than in Achenbach's case particularly at smaller ratios of d_p/d.

NOTATION

C_D = drag coefficient
C_D' = drag coefficient defined by equation (1)
$C_{D\infty}$ = drag coefficient in an infinite fluid
d = tube diameter
d_p = disk or sphere diameter
d_a = diameter of supporting axle
g = acceleration due to gravity

L = length of the working tube
p = pressure
w = mean velocity of fluid in the tube
w_r = mean velocity in the smallest annular cross-section
γ = specific weight of the fluid
λ = coefficient of longitudinal friction
μ = viscosity
ρ = density

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Adsorption Rates for Sulfur Dioxide and Hydrogen Sulfide in Beds of Activated Carbon

JULIEN ANDRIEU

and

J. M. SMITH

University of California
Davis, CA 95616

Beds of activated carbon are useful for separating carbon dioxide, sulfur dioxide, and hydrogen sulfide from air and combustion gases. For *a priori* design of adsorption beds, rate and equilibrium (for reversible adsorption) parameters need to be known. The usual model for the overall process includes four steps. These steps (and their rate constants) are: axial dispersion (E_d), gas-to-particle mass transfer (k_f), intraparticle diffusion (D_n, D_i), and adsorption (k) at a site within the pores of the carbon particle. Since the adsorption step corresponds to physical adsorption, its intrinsic rate is fast enough that this effect on the overall process is negligible. Gas-to-particle mass transport also is a relatively rapid step so that k_f can be estimated accurately enough from available correlations (e.g., Wakao and Funazkri, 1978). In commercial-scale columns, axial dispersion is usually insignificant. However, E_d can be estimated from correlations (for example, Suzuki and Smith, 1972). Hence, intraparticle diffusivities in the macro- and micropore regions and the adsorption equilibrium constant K_A are the parameters that need to be determined from laboratory studies.

Andrieu and Smith (1979) have reported values of these parameters for adsorption of carbon dioxide on a common type (BPL, Calgon Corp.) of activated carbon, prepared by binding microparticles of bituminous coal prior to activation. Pulse-response data were analyzed by the method of moments to obtain K_A, D_n and D_i. The method of analysis, including the accounting for axial dispersion, which is significant in laboratory-scale experiments, is described in the carbon dioxide work.

The purpose of the present paper is to report results for sulfur dioxide and hydrogen sulfide for the same activated carbon. The experimental apparatus, procedure, and method of analysis were the same as employed for the carbon dioxide study. Hence, only the results and comparison for the three adsorbates are given here. More information (Calgon Corp.) is now available for the pore-volume distribution for BPL carbon, and the distribution curve is shown in Figure 1. Pore sizes cover a wide range from an average value of 11Å for micropores (diameter $\leq$ 50Å) and 3750Å (diameter $\geq$ 50Å) for macropores. As for carbon dioxide, reasonable values of the diffusivities, and their temperature dependency, could be obtained by dividing the pore volume into micropores and macropores. Mass transfer in the macropores was assumed to occur by bulk and Knudsen diffusion while an activated surface process was assumed for the small micropores, where molecule-wall interaction predominates.

ADSORPTION EQUILIBRIUM RESULTS

Equilibrium constants are obtained from corrected first moments, ($\Delta\mu_1$), evaluated from the measured response curves. Figure 2 illustrates such data as a function of gas velocity through the bed. The agreement between the data points for different particle sizes is a measure of the reproducibility and accuracy of the measurements.

Equilibrium constants K_A and isoteric heats of adsorption ΔH_A are summarized in Table 1. The temperature range studied for each adsorbate was chosen so as to give rapidly reversible adsorption; the response curves did not have excessively long tails. The K_A values are a measure of the adsorption strength, and the values in the table show that the capacity increases from CO_2 to H_2S to SO_2. The results for the same temperature (343°K) suggest relative capacities of 1.0 (CO_2), 4.8 (H_2S) and 51 (SO_2). The heats of adsorption, which apply at essentially zero surface coverage since the data were obtained chromatographically, increase in the same order. The values for H_2S and SO_2 are about 60% greater than their heats of condensation (at the normal boiling point), while the adsorption and sublimation heats are about the same for carbon dioxide. These results suggest stronger molecule-carbon site interaction for the sulfur gases than for carbon dioxide.

INTRAPARTICLE DIFFUSIVITIES

The results for macropore diffusion, D_n and τ_n, are given in Table 2. The values for the diffusivity for the three absorbates

J. Andrieu is on leave from National Institute of Applied Science, Lyon, France.

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